

Combination, Permutation, Probability (2)

1. Find the number of permutations that can be formed from the letters of the word POPULAR. How many of these permutations:
 - (a) begin and end with P?
 - (b) have the two P's separated?
 - (c) have the vowels together?
2. A box contains 5 green marbles, 4 blue marbles and 6 red marbles. A marble is picked at random. Without replacing the first marble, another marble is taken from the box. Calculate the probability that
 - (a) the first marble is green and the second marble red.
 - (b) two marbles are NOT of the same colour.
3. Events A and B are events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{4}$ and $P(A \text{ or } B \text{ but not both}) = \frac{1}{3}$.
 - (a) Find (i) $P(A \cap B)$ (ii) $P(A' \cap B)$ (iii) $P(A|B)$ (iv) $P(B|A')$
 - (b) State, with explanations, whether A and B are
 - (i) independent
 - (ii) mutually exclusive.
4. A boy plays a computer game in which he has to drive successfully a vehicle through a terrain within a certain time. In the first attempt the probability he can succeed is 0.75 and in subsequent attempts if he is successful the difficulty increases and the probability of success is half the probability of his previous attempt. If he is unsuccessful the probability remains the same. He plays three games.
 - (a) Find the probability he is successful in all three games.
 - (b) Given that he is successful in the first game, find the probability he is successful in exactly two games.
5. Three red cards are labeled A, B and C, and seven blue cards are labeled 1, 2, 3, 4, 5, 6 and 7. Find the number of ways we can
 - (a) arrange all the cards in a straight line as that the cards of the same colour are next to each other,
 - (b) choose and arrange equal number of red and blue cards in a straight line so that the cards of the same colour are next to each other.
6.
 - (a) Given that digits 2, 3, 4, 5, 6, three digits numbers with values greater than 400 are formed. In how many ways can this be done if the numbers formed are odd and if the digits formed may not be used repeatedly.
 - (b) The digits of the numbers 421265 are arranged so that the resulting number is even. Find the number of ways in which this can be done.

7. Ada and Bill are shooting at a target. The probability that Ada's shot hits the target is $\frac{1}{2}$ and the probability that Bill's shot misses the target is $\frac{1}{3}$. What is the probability that:
- both their shots hit the target?
 - only one of their shots hits the target?
 - none of their shots hits the target?
8. The events A and B are such that $P(B) = 0.4$, $P(A'|B) = 0.8$ and $P(B|A') = 0.5$, find
- $P(A \cap B)$
 - $P(A \cup B)$
9. A and B play 8 games of tennis, 4 on Saturday and 4 on Sunday.
- Find the number of ways A can win 2 games on Saturday and 2 on Sunday.
 - Find the number of ways A can win a total of 4 games over 2 days.
 - Show that $\binom{8}{4} = \binom{4}{0}^2 + \binom{4}{1}^2 + \binom{4}{2}^2 + \binom{4}{3}^2 + \binom{4}{4}^2$
10. A boy wishes to buy exactly six marbles. There are four different colours of marbles available. In how many ways can he buy the six marbles. (Hint: Arrange 111000000)
11. Ten items were taken from a large population where the probability of an item is defective 0.02.
- What is the probability that no defective items are taken?
 - What is the probability that at most one item is defective?
12. Experience shows that 40% of the throws of a basketball result in enter the ring.
- Find the probability, out of ten throws made by the basketballer, at least two throws result in entering the ring.
 - Find the probability that at most five throws need to be made by the basketballer so that four throws result in entering the ring.
13. A type of seed is sold in packets which contain ten seeds each. On the average, it is found that a seed per packet does not germinate. Find the probability that a packet chosen at random contains less than two seeds which does not germinate.